Math 55 Quiz 3 DIS 106

Name: ________________________________ 14 Feb 2022

1. Prove or disprove each of the following statements:

   (a) The function $f : \mathbb{Q} \to \mathbb{Q}$ defined by $f(x) = x^2$ is onto. [3 points]

   (b) If f is an injective function from A to B, and S is a subset of A, then $f^{-1}(f(S)) = S$. [4 points]

   (c) Suppose a, b, c, d are positive integers. If $a \equiv b \pmod 2$ and $c \equiv d \pmod{14}$, then $ab \equiv cd \pmod{28}$. [3 points]

(a) This is false. We claim that -1 is not mapped to by any element of $\mathbb{Q}$:
Suppose otherwise, then $f(x) = x^2 = -1$ for some $x \in \mathbb{Q}$. But then $-1 = x^2 \geq 0$; contradiction.

(b) This is true.
For any $x \in f^{-1}(f(S))$, $f(x) \in f(S)$, hence there exists $x' \in S$ such that $f(x) = f(x')$.
Since f is injective, $x = x'$. This implies that $x = x' \in S$. So $f^{-1}(f(S)) \subseteq S$
Conversely, for any $x \in S$, $f(x) \in f(S)$, hence $x \in f^{-1}(f(S))$. So $S \subseteq f^{-1}(f(S))$.
Hence $f^{-1}(f(S)) = S$.

(c) This is false. For example, take $a = 1, b = 1, c = 1, d = 15$, then $a \equiv b \pmod 2$ and $c \equiv d \pmod{14}$ but $ab = 1 \not\equiv cd = 15 \pmod{28}$.